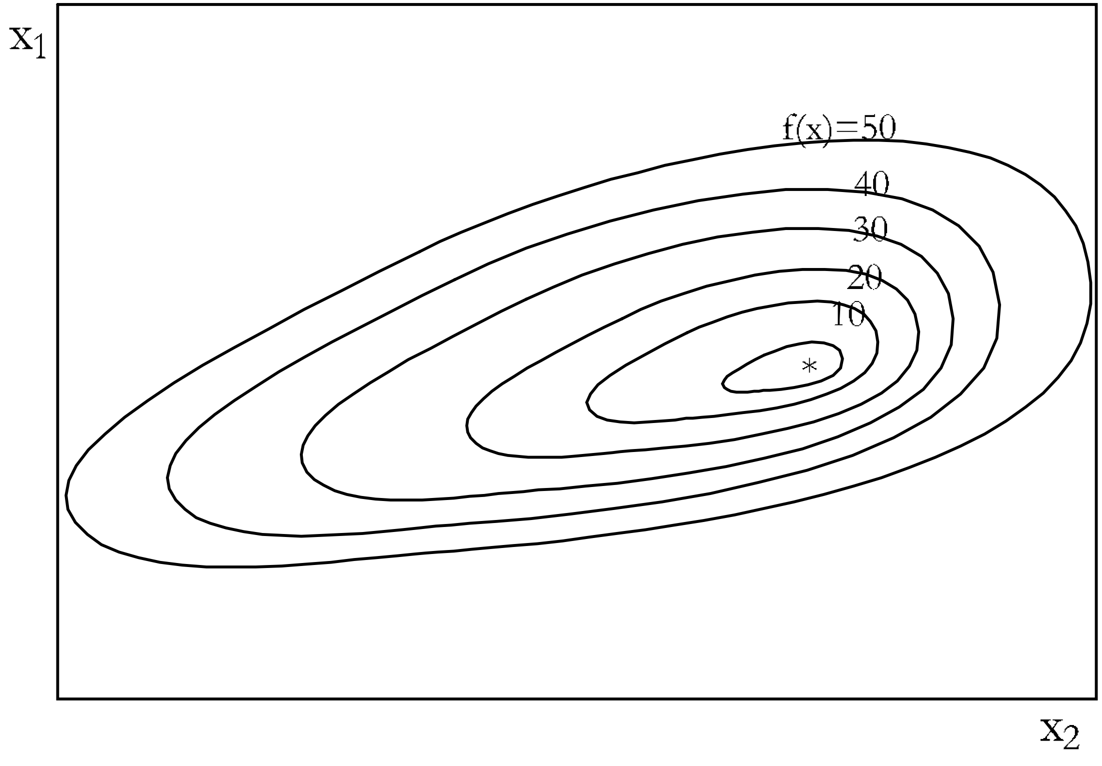
Non-linear Programming

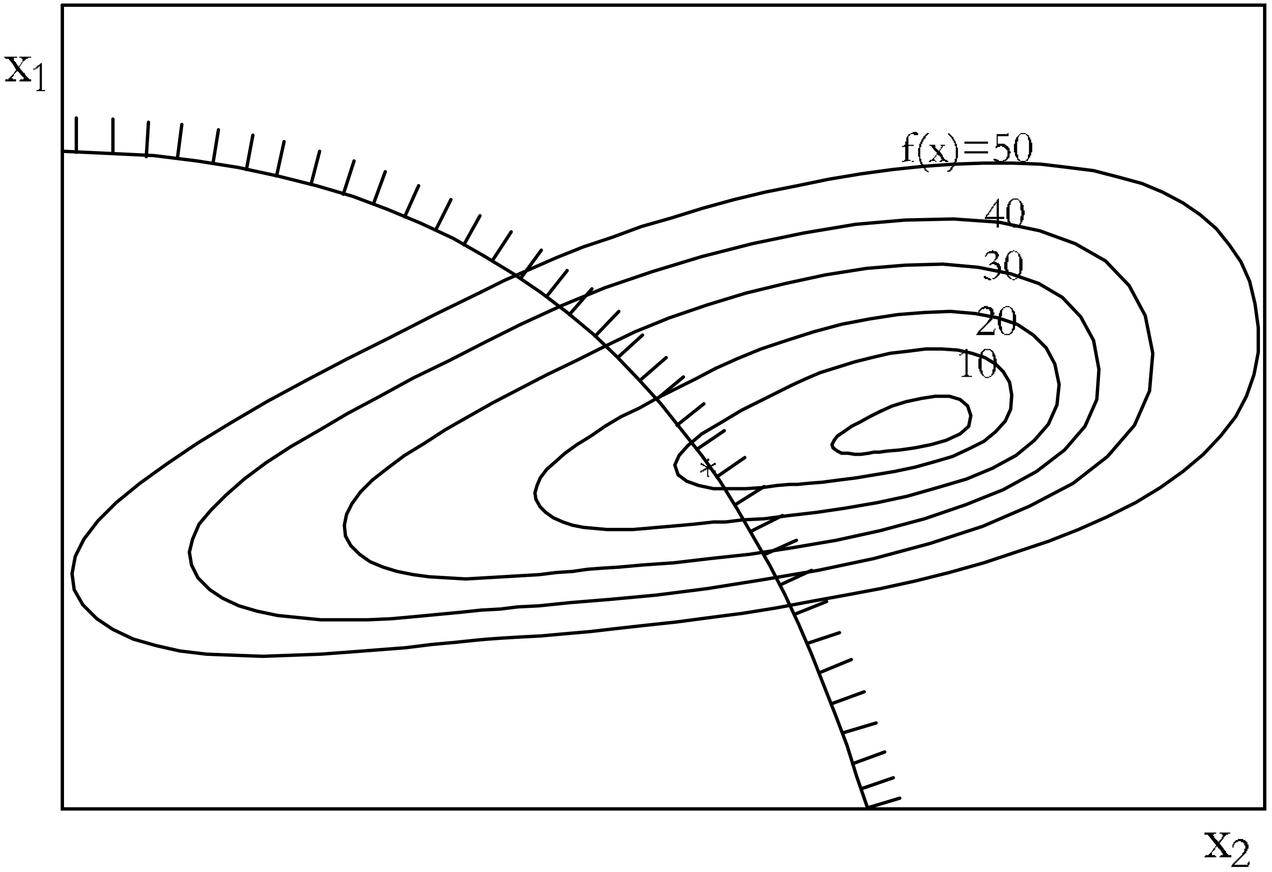
The unconstrained optimisation problem, with an objective function and no constraints, can be illustrated with height curves when the problem has two variables *x*1 and *x*2. The optimal solution is depicted with a *

A ball, placed anywhere, will eventually roll down to the optimal solution.



Non-linear Programming

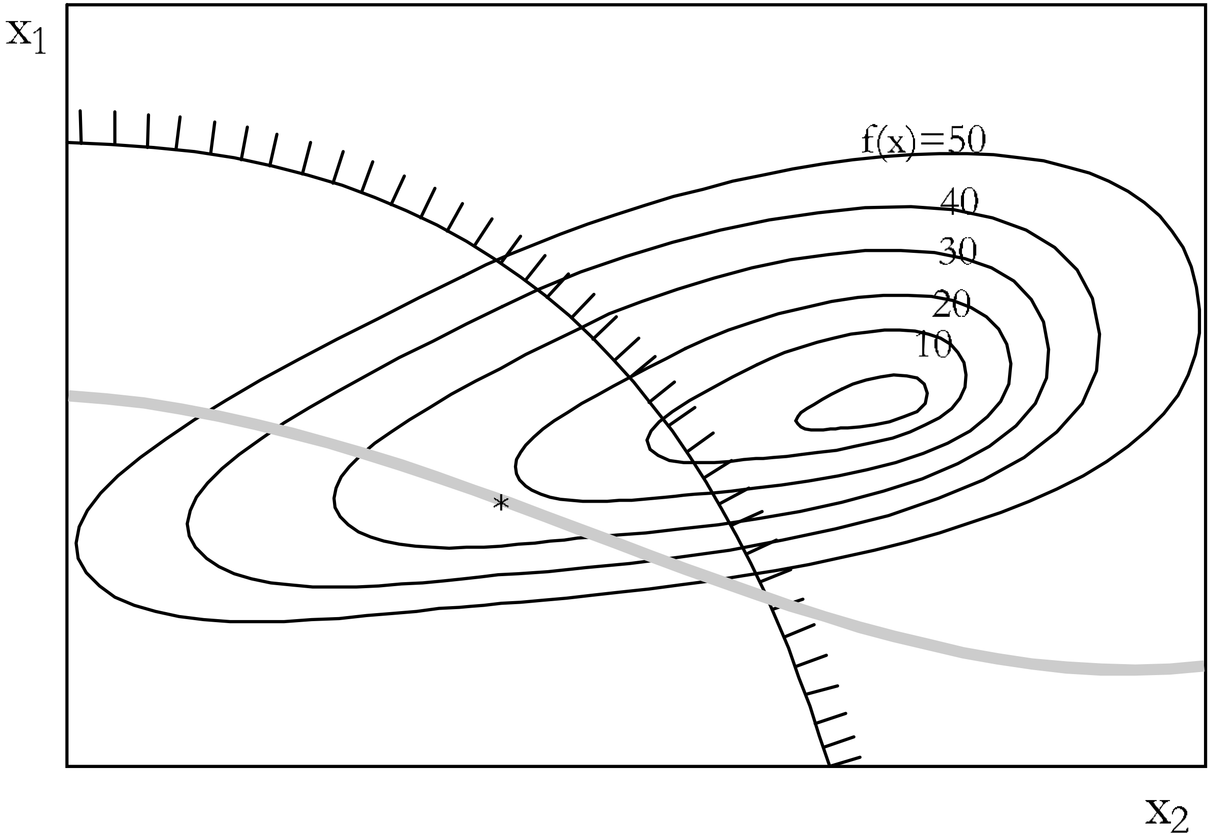
The constrained problem has, besides an objective function, equality and inequality constraints restricting the feasible region. In Figure an inequality constraint has been added to the problem.



Non-linear Programming

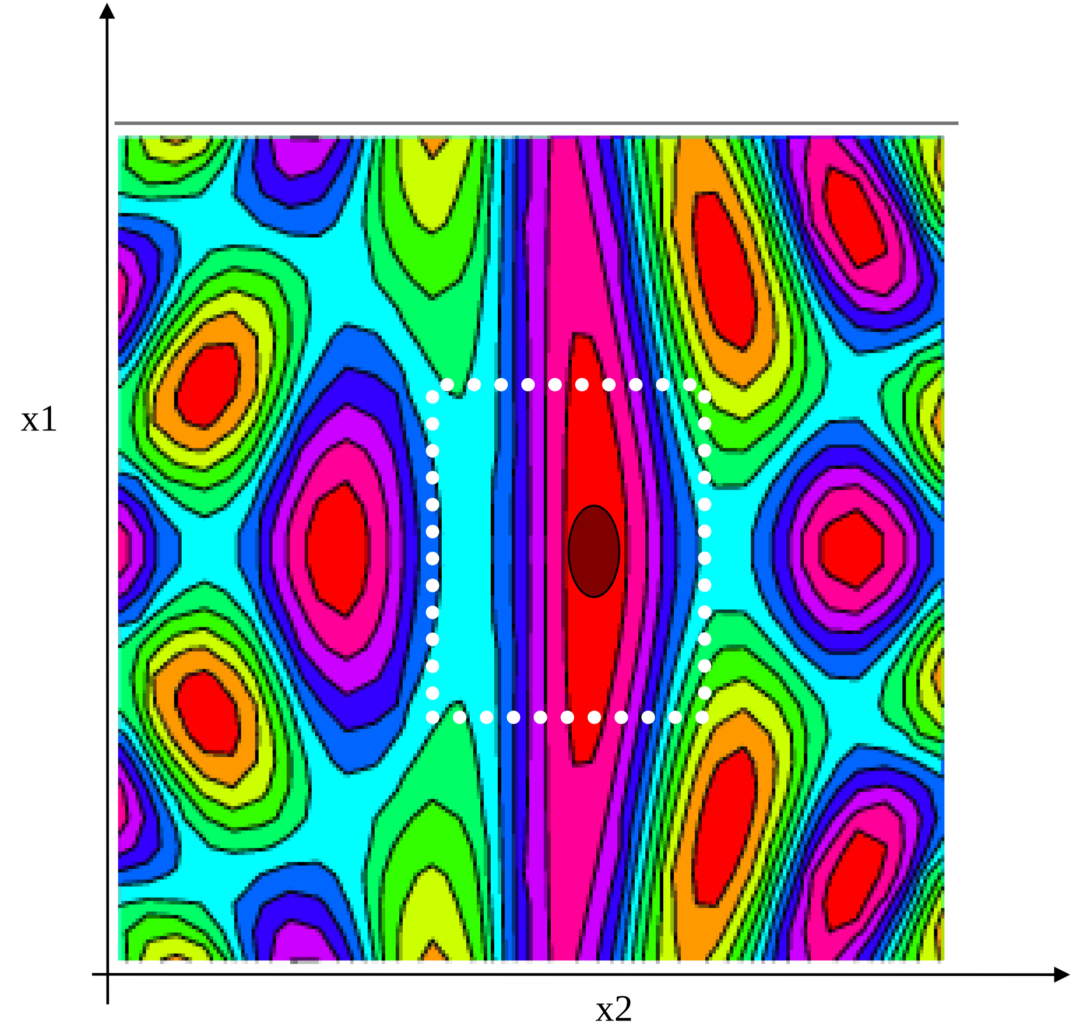
The inequality constraint can bee seen as a fence preventing the ball from rolling further down.

The equality constraint can be seen as a railroad forcing the ball to follow the track.



Initial point!

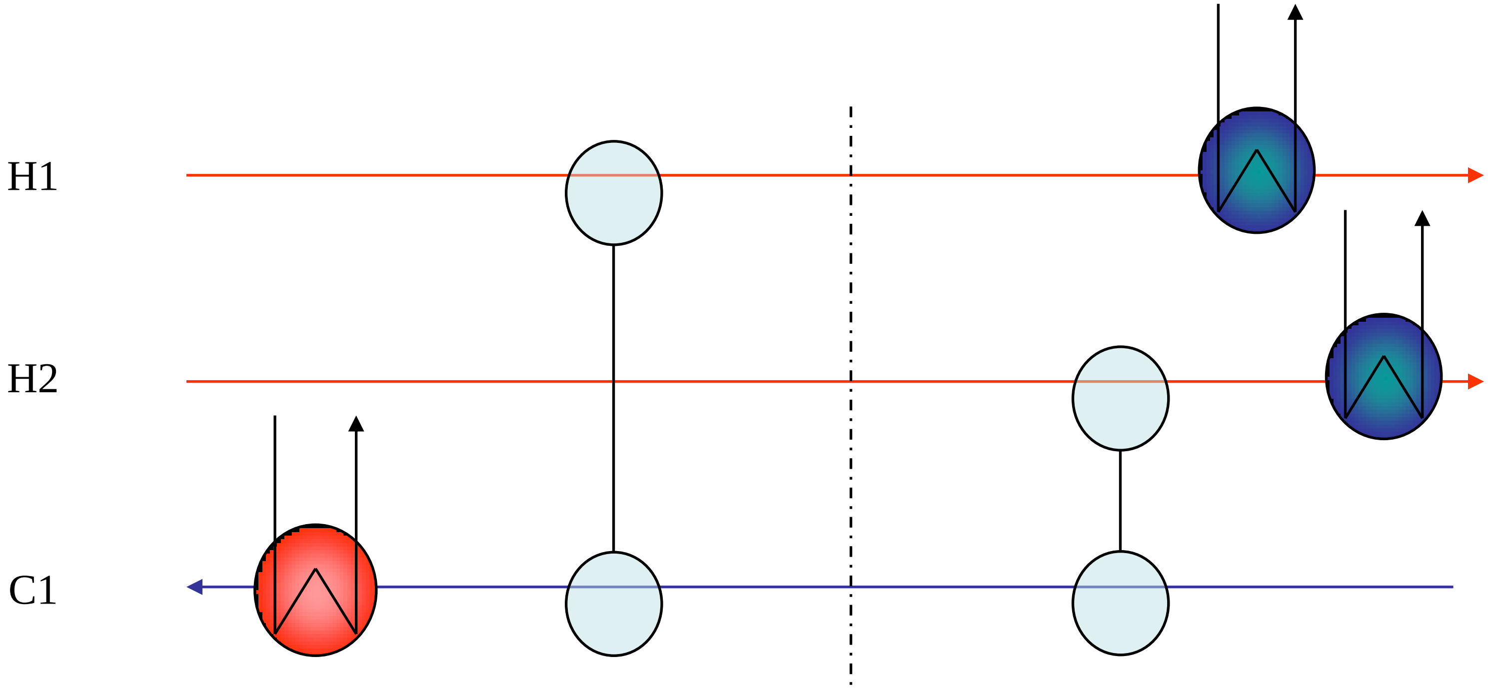
Bounds!



Minimum costs HEN

The second solution of the MILP transshipment model

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR Z	-INF	5.000	+INF	.
---- VAR QS11	.	62.500	+INF	.
---- VAR QS12	.	112.500	+INF	.
---- VAR QS13	.	.	+INF	EPS
---- VAR Q1W3	.	25.000	+INF	.
---- VAR Q2W3	.	50.000	+INF	.
---- VAR Q112	.	75.000	+INF	.
---- VAR Q113	.	.	+INF	EPS
---- VAR Q213	.	250.000	+INF	.
---- VAR RS1	.	112.500	+INF	.
---- VAR RS2	.	.	+INF	EPS
---- VAR R12	.	.	+INF	EPS
---- VAR yas1	.	1.000	1.000	1.000
---- VAR ya11	.	1.000	1.000	1.000
---- VAR yb11	.	.	1.000	1.000
---- VAR yb21	.	1.000	1.000	1.000
---- VAR ybw1	.	1.000	1.000	1.000
---- VAR ybw2	.	1.000	1.000	1.000


H1
H2
C1

Mathematical formulation of HEN superstructure

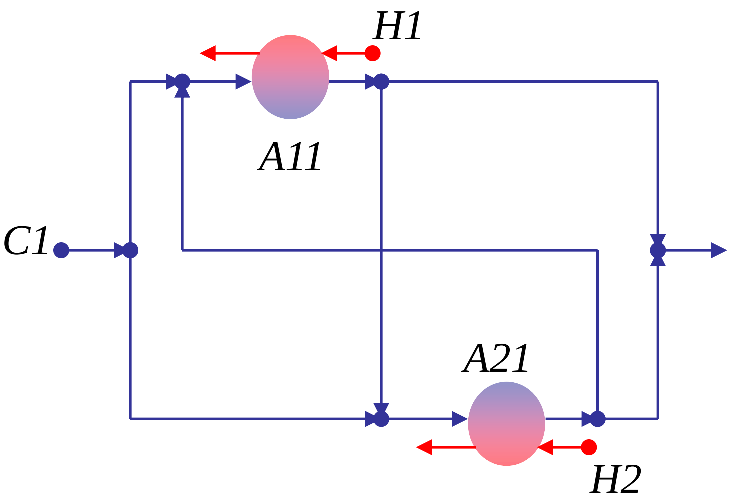
Given the:

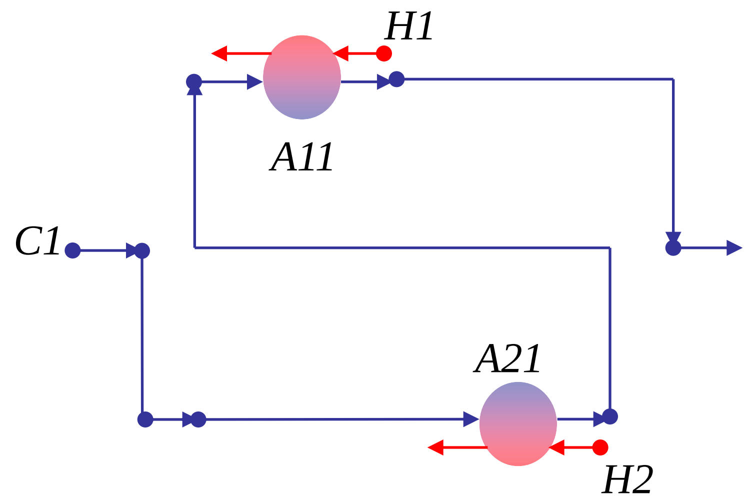
- Minimum loads of hot and cold utilities (LP) (assume fixed matches)
- Location of pinch points (LP)
- Minimum number of matches in network (MILP)
- Heat loads of each match in network (MILP)

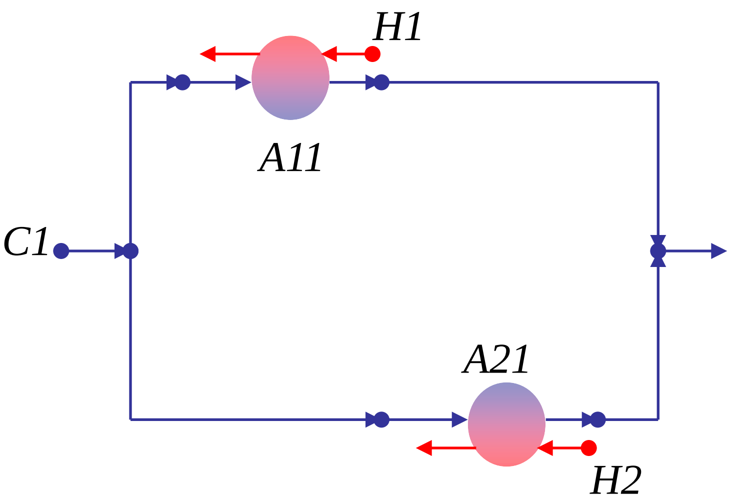
Draw the superstructure and define the variables and the given parameters, see figure 16.14 and the variable list below (page 549).

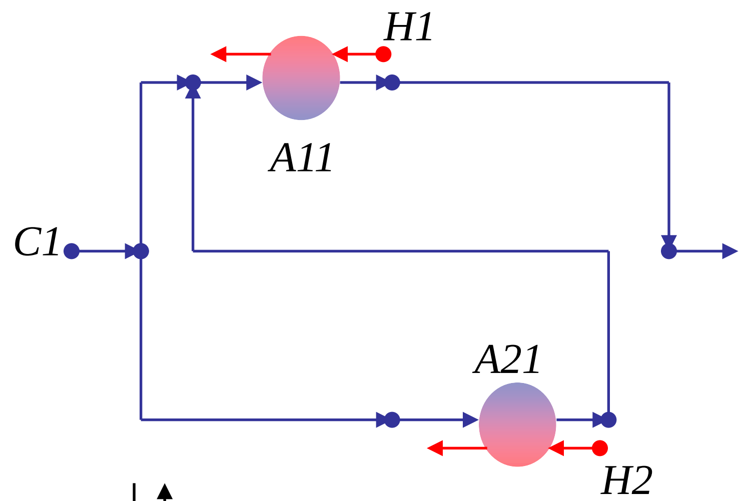
Use the second solution of the MILP transshipment model.

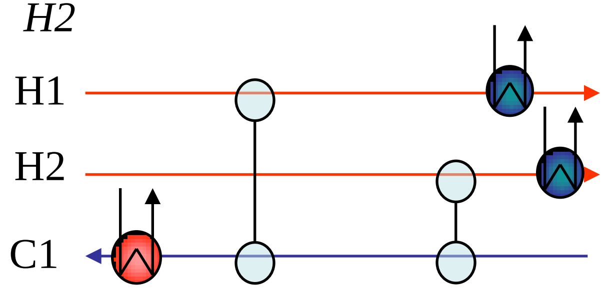
HEN superstructure







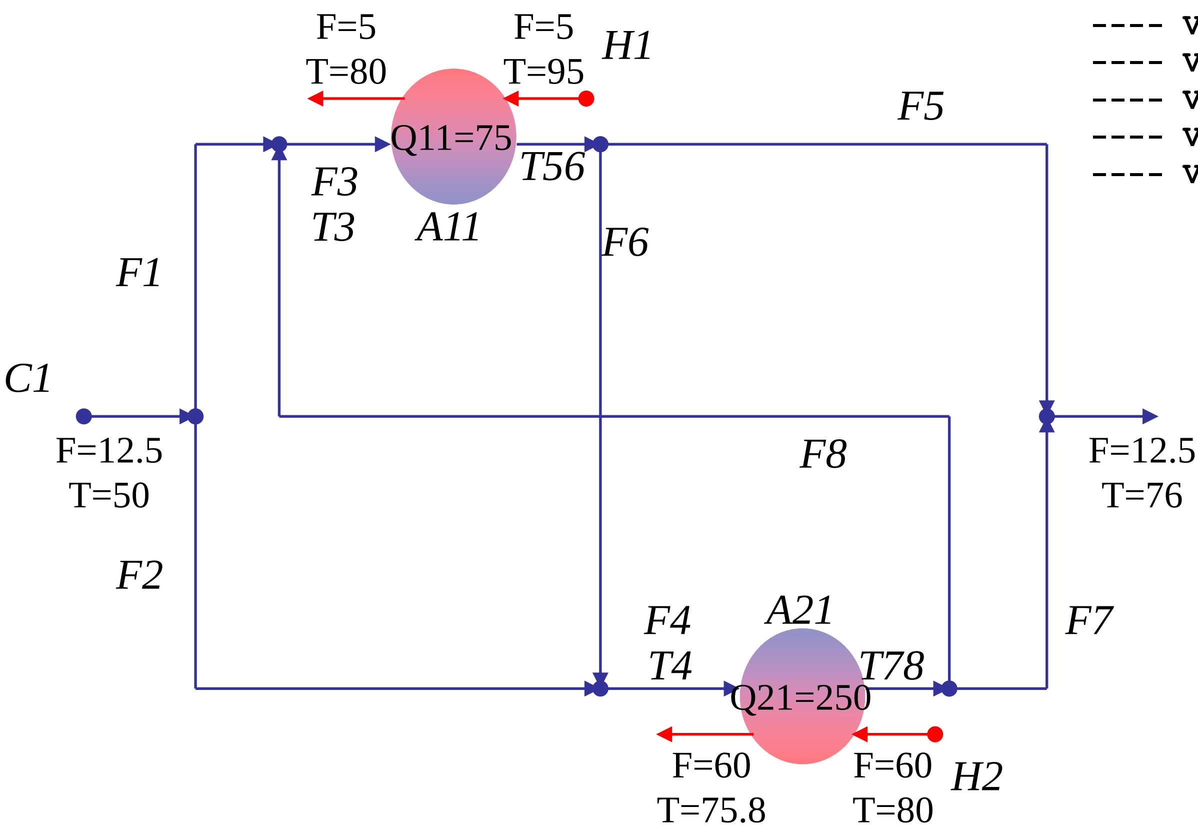




Variables and HEN superstructure

Normally we should design two HEN:s, below and above the pinch.
Now, only one HEN is considered to learn the idea of superstructure.

	LOWER	LEVEL
---- VAR Z	-INF	5.000
---- VAR QS11	.	62.500
---- VAR QS12	.	112.500
---- VAR QS13	.	.
---- VAR Q1W3	.	25.000
---- VAR Q2W3	.	50.000
---- VAR Q112	.	75.000
---- VAR Q113	.	.
---- VAR Q213	.	250.000
---- VAR RS1	.	112.500
---- VAR RS2	.	.
---- VAR R12	.	.
---- VAR yas1	.	1.000
---- VAR ya11	.	1.000
---- VAR yb11	.	.
---- VAR yb21	.	1.000
---- VAR ybw1	.	1.000
---- VAR ybw2	.	1.000



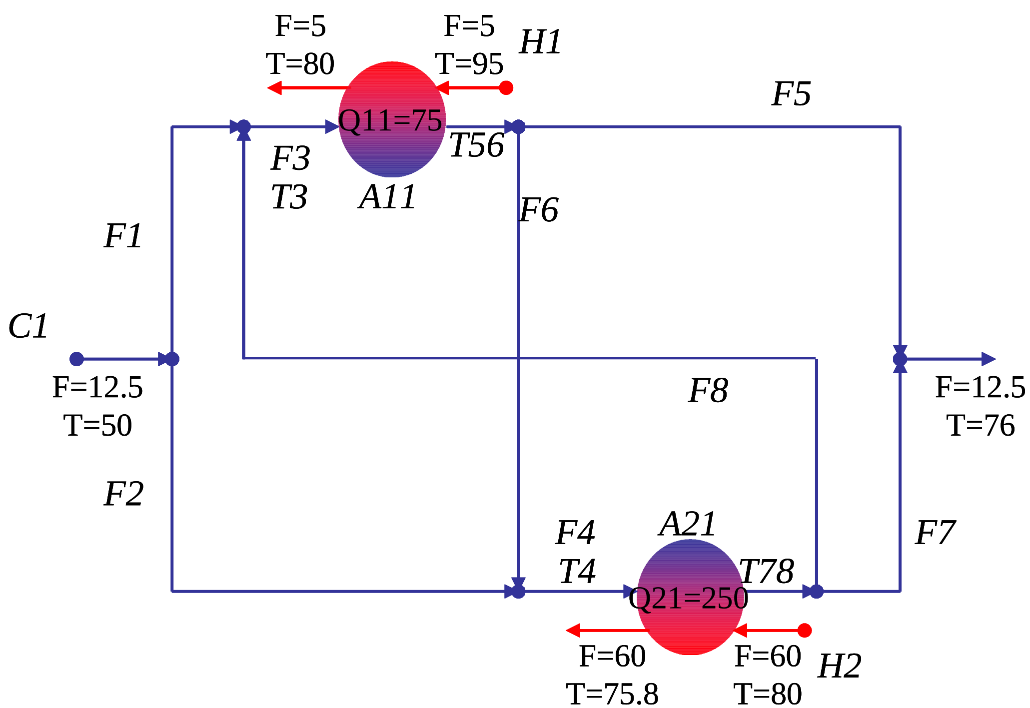
Variables and HEN superstructure

VARIABLES Z Minimum area cost ;

POSITIVE VARIABLES

F1, F2, F3, F4, F5, F6, F7, F8 flow rate heat capacities

T3, T4, T56, T78 temperatures;



Mass balances and HEN superstructure

* Mass balances for splitters

MBS1.. F1+F2=E=12.5;

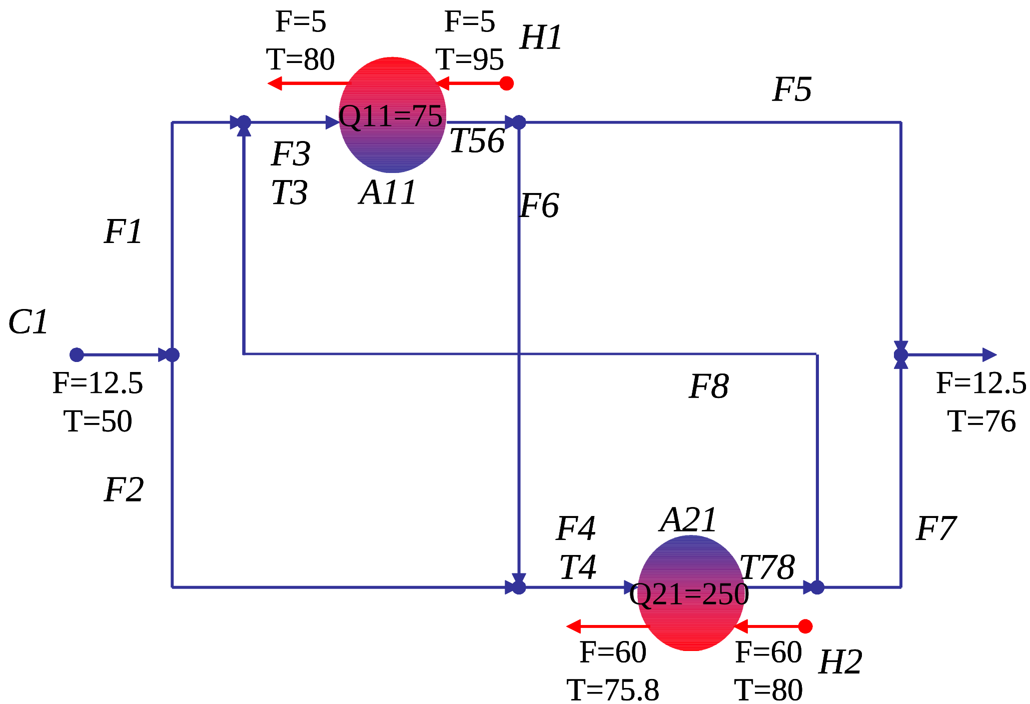
MBS2.. F3-F5-F6=E=0;

MBS3.. F4-F7-F8=E=0;

* Mass balances for mixers

MBM1.. F3-F1-F8=E=0;

MBM2.. F4-F2-F6=E=0;



Energy balances and HEN superstructure

*** Energy balances for mixers**

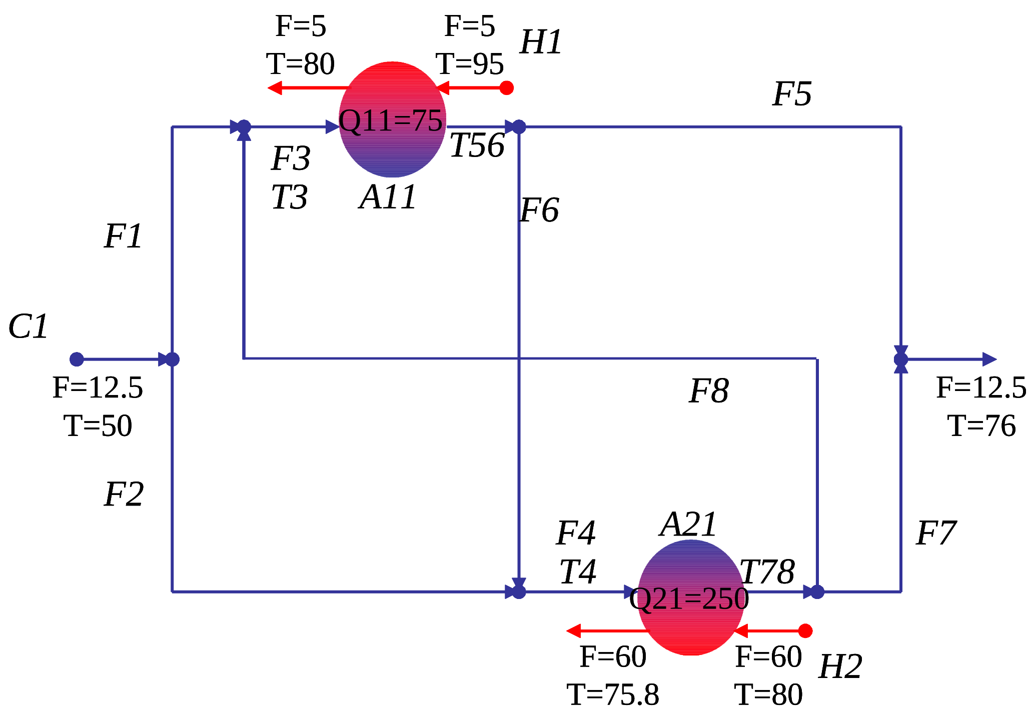
EBM1.. F1*50+F8*T78-F3*T3=E=0;

EBM2.. F2*50+F6*T56-F4*T4=E=0;

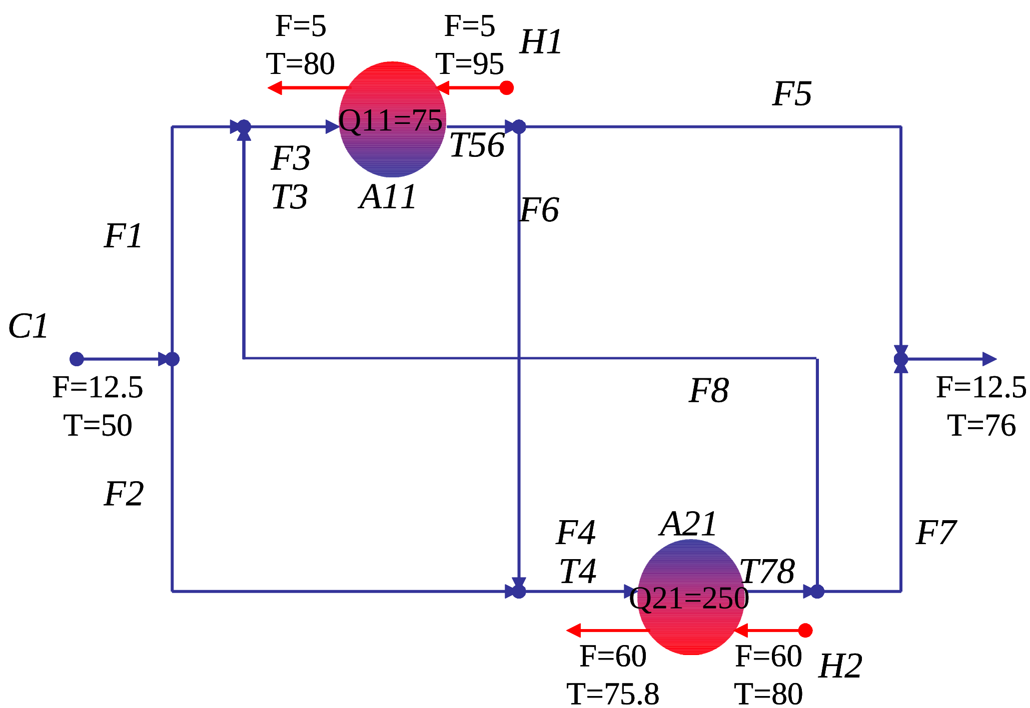
*** Energy balances in exchangers**

EBE1.. F3*(T56-T3)=E=75;

EBE2.. F4*(T78-T4)=E=250;



Feasibility constraints and HEN superstructure



***Feasibility constraints**

FC11.. T3=G=50;

FC12.. T3=L=80-EMAT;

FC21.. T56=G=50+(75/12.5);

FC22.. T56=L=95-EMAT;

FC31.. T4=G=50;

FC32.. T4=L=75.83-EMAT;

FC41.. T78=G=50+(250/12.5);

FC42.. T78=L=80-EMAT;

FC51.. EMAT=G=0;

The Paterson approximation is used to calculate the log mean temperature difference (LMTD) to avoid numerical difficulties:

$$LMTD = \frac{(th_{in} - tc_{out}) - (th_{out} - tc_{in})}{\ln \dfrac{th_{in} - tc_{out}}{th_{out} - tc_{in}}}$$

$$LMTDp_{i,j} = \frac{2}{3}\left[(th_{out} - tc_{in})(th_{in} - tc_{out})\right]^{\frac{1}{2}} + \frac{(th_{out} - tc_{in}) + (th_{in} - tc_{out})}{6}$$

Objective function:

$$\min\ Z = C1 \times \left(\frac{Q_{11}}{\left(\dfrac{1}{h_{c1}} + \dfrac{1}{h_{h1}}\right)^{-1} \times LMTDp_{11}} \right)^{0.6} + C2 \times \left(\frac{Q_{21}}{\left(\dfrac{1}{h_{c1}} + \dfrac{1}{h_{h2}}\right)^{-1} \times LMTDp_{21}} \right)^{0.6}$$

Providing upper and lower bounds on all optimization variables is necessary to avoid numerical difficulties.

Formulate the NLP model, see page 550.

Objective function and HEN superstructure

Type	Tstart (C)	Ttarget (C)	FCp (kW/K)	Q (kW)	h (kW/m²K)
Hot	95	75	5	100	2
Hot	80	75	60	300	0.6667
Cold	50	90	12.5	500	2

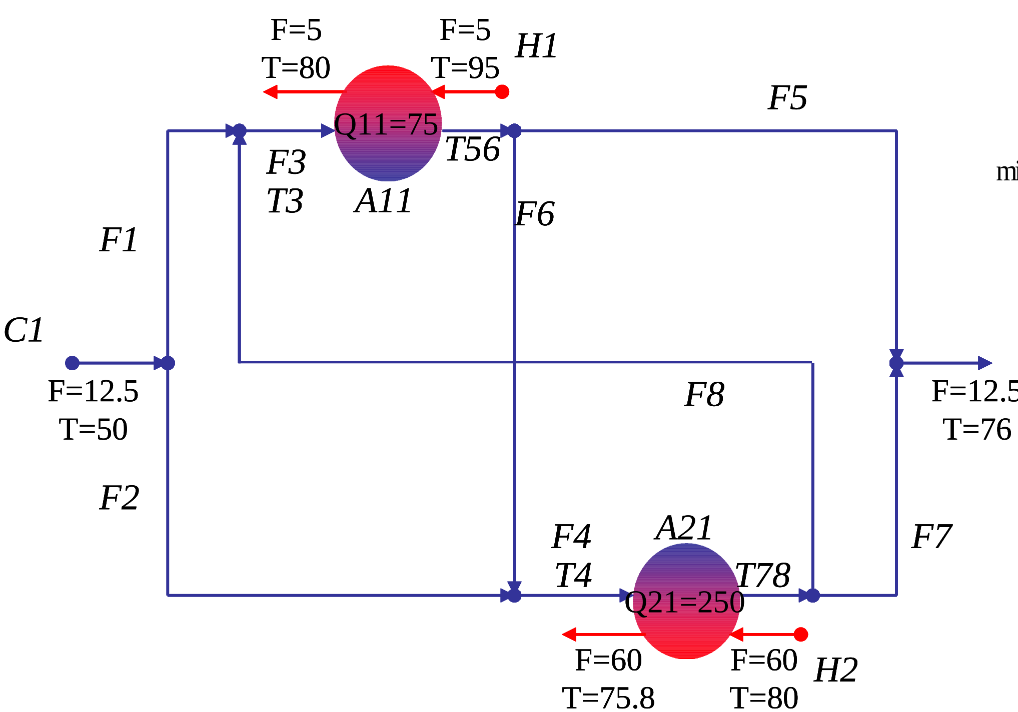
Cost of heat exchanger = 1300 $\text{Area}^{0.6}$ €/Yr; Area in m^2

HRAT = 10 K; EMAT $\geq$ 0 K

OBJ.. Z =E=

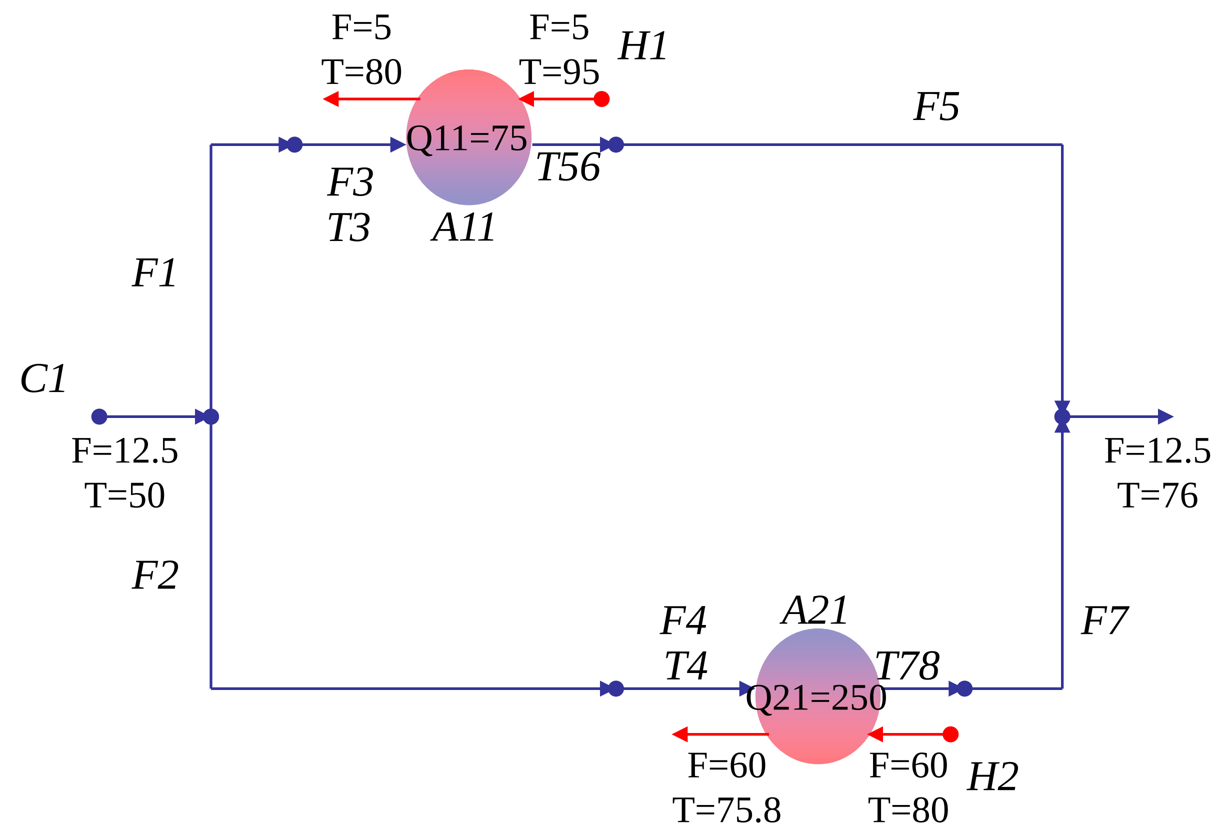
1300*(75/(1*(2/3*((80-T3)*(95-T56))(0.5)+((80-T3) + (95-T56))/6)))**(0.6) +**

1300*(250/(0.5*(2/3*((75.83-T4)*(80-T78))(0.5)+((75.83-T4) + (80-T78))/6)))**(0.6) ;**



$$\min\ Z = C1 \times \left(\frac{Q_{11}}{\left(\frac{1}{h_{c1}} + \frac{1}{h_{h1}}\right)^{-1} \times LMTDp_{11}} \right)^{0.6} + C2 \times \left(\frac{Q_{21}}{\left(\frac{1}{h_{c1}} + \frac{1}{h_{h2}}\right)^{-1} \times LMTDp_{21}} \right)^{0.6}$$

Variables and HEN superstructure

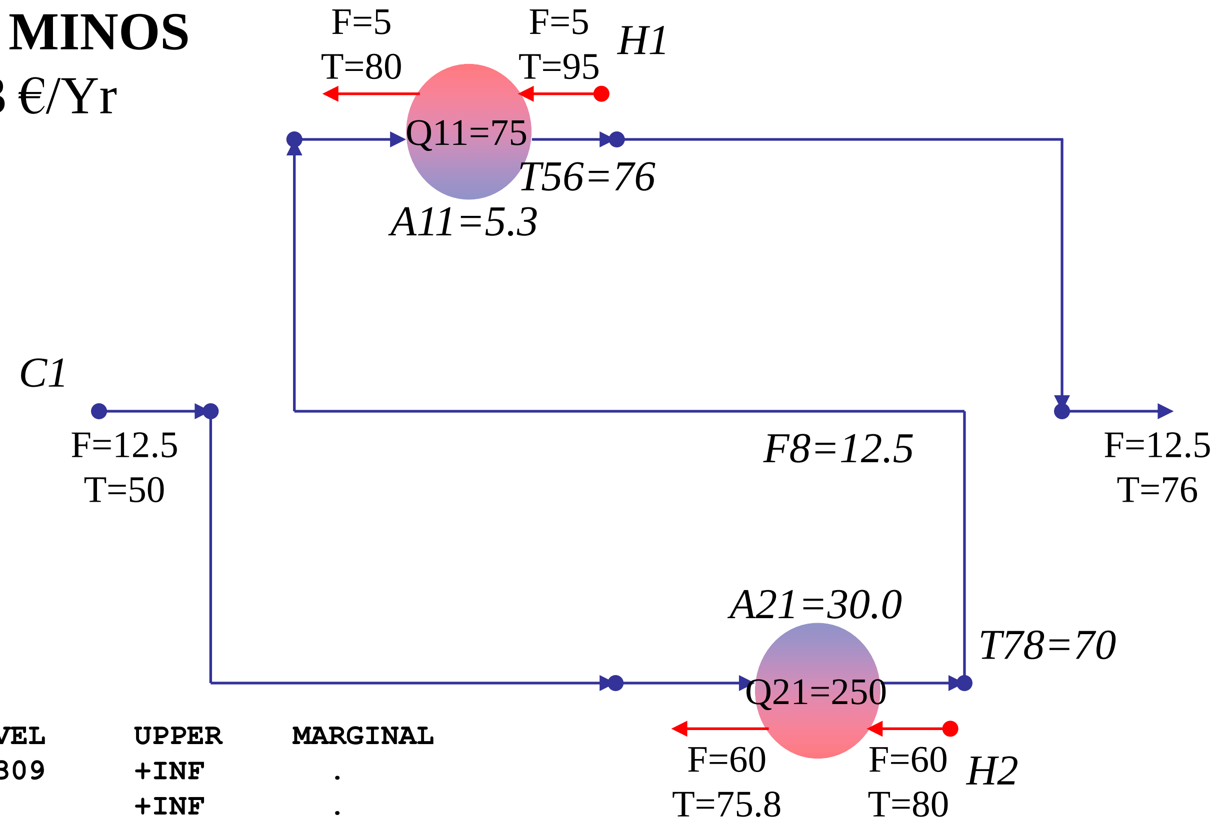


---- VAR Z	-INF	14394.437
---- VAR F1	.	2.199
---- VAR F2	.	10.301
---- VAR F3	.	2.199
---- VAR F4	.	10.301
---- VAR F5	.	2.199
---- VAR F6	.	.
---- VAR F7	.	10.301
---- VAR F8	.	.
---- VAR T3	.	50.000
---- VAR T4	.	50.000
---- VAR T56	.	84.102
---- VAR T78	.	74.270

PARAMETER Area1 = 3.974
PARAMETER Area2 = 37.396

Final solution with MINOS

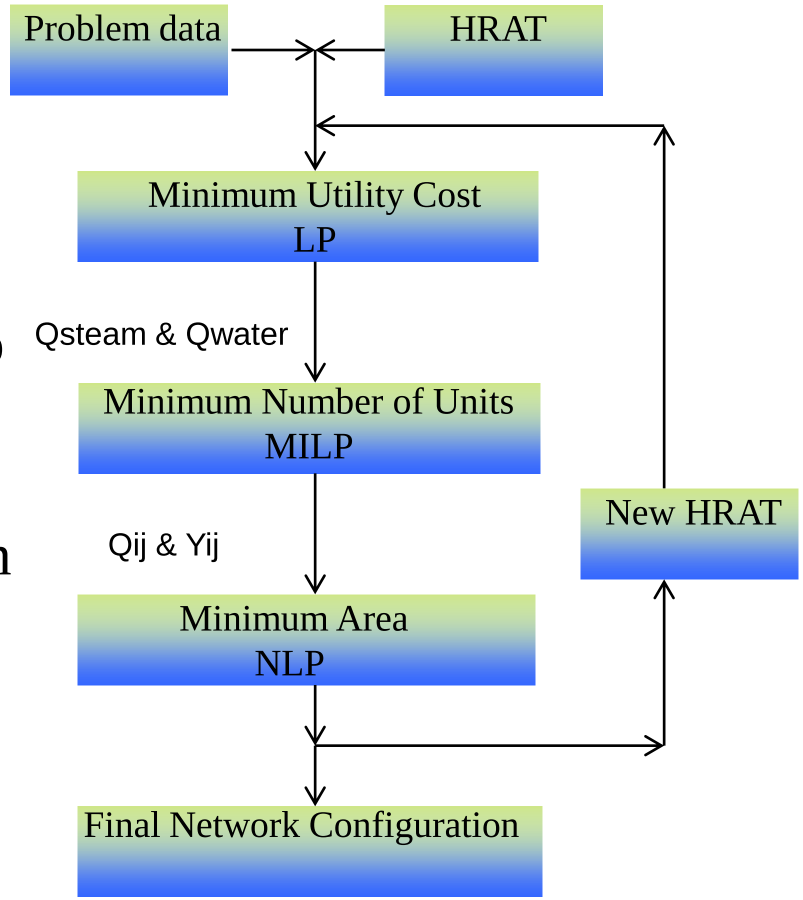
Investment 13553.3 €/Yr



	LOWER	LEVEL	UPPER	MARGINAL
---- VAR Z	-INF	13553.309	+INF	.
---- VAR F1	.	.	+INF	.
---- VAR F2	.	12.500	+INF	.
---- VAR F3	.	12.500	+INF	.
---- VAR F4	.	12.500	+INF	.
---- VAR F5	.	12.500	+INF	.
---- VAR F6	.	.	+INF	.
---- VAR F7	.	.	+INF	29.828
---- VAR F8	.	12.500	+INF	.
---- VAR T3	.	70.000	+INF	.
---- VAR T4	.	50.000	+INF	.
---- VAR T56	.	76.000	+INF	.
---- VAR T78	.	70.000	+INF	.
---- VAR EMAT	.	10.000	+INF	.

```
PARAMETER Area1          =        5.348
PARAMETER Area2          =       29.965
```

HEN synthesis strategy

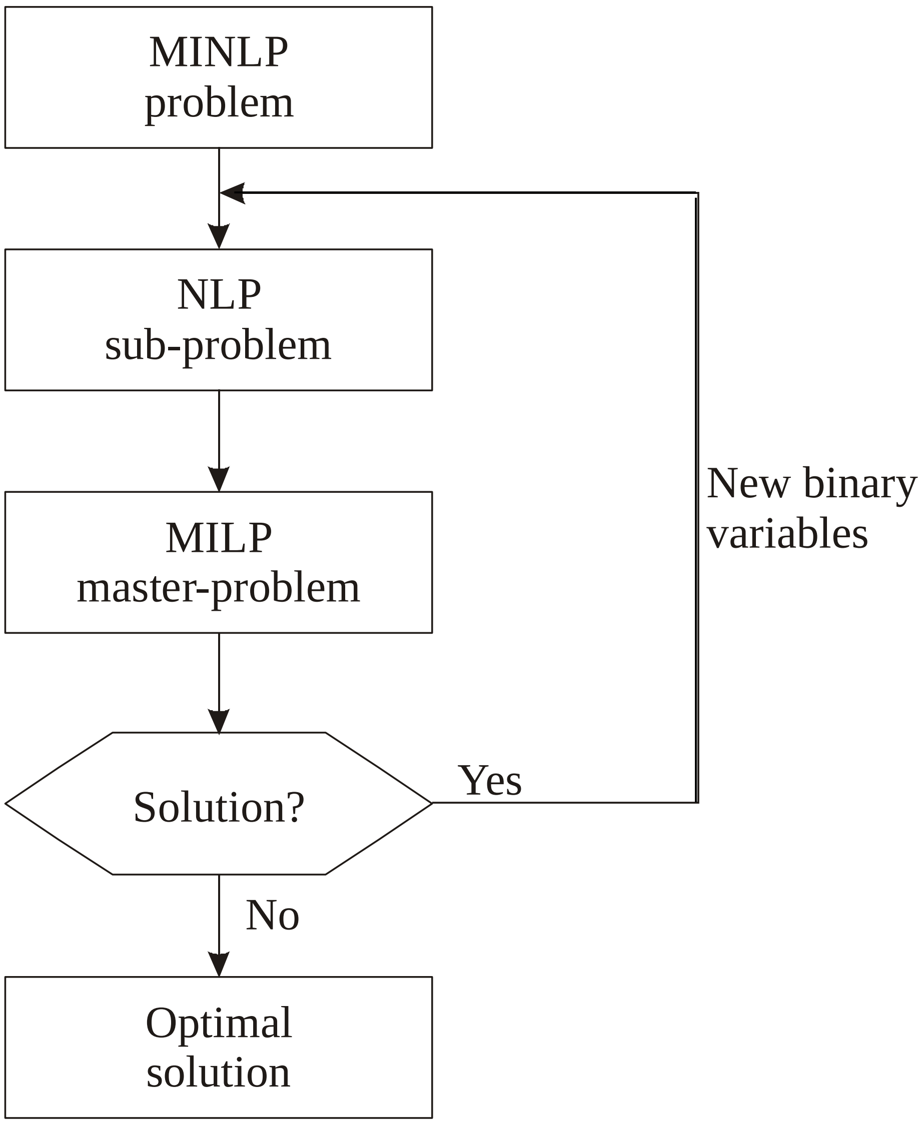


HEN synthesis strategy developed by Floudas et al. (1986), is based on decomposition-based method that is also called sequential method.

The HRAT is the only fixed parameter in the three sequential stages and can subsequently be updated by performing some search algorithm.

This HEN synthesis strategy must be applied for all global solutions of minimum number of units.

Principles of the MINLP methods (OA)



The NLP sub-problem solves for the optimal continuous variables when the binary variables are held fixed and results in an upper bound for the solution.

The master problem selects of the remaining integer combinations the one with the lowest lower bound. When the lower bound exceeds the upper bound the algorithm is terminated.

The MILP master problem involves all linear constraints from the original MINLP problem and the linearisations obtained from the NLP sub-problem.

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http://agecon.tamu.edu/faculty/mccarl/regbook.htm

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